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**Algorithm 1** Prim's algorithm with edge priority queue

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1: Input: Graph  $G = (V, E)$ 
2: Output: Minimum spanning tree of  $G$ 
3:
4: // Initialization
5: Select an arbitrary vertex  $s \in V$ 
6: Construct a tree  $mst$  that contains only the vertex  $s$ 
7: Construct an empty priority queue  $Q$  that will contain edges ordered by their weights // We
   will maintain the invariant that for every edge  $(u, v) \in Q$ , at least one of  $u$  or  $v$  is in  $mst$ 
8: for  $v \in \text{neighbors}(s)$  do
9:   Add edge  $(v, s)$  to  $Q$ 
10: end for
11:
12: // Construct  $mst$ 
13: while  $Q$  is not empty do
14:   Let edge  $(u, v) = Q.\text{findMin}()$ 
15:    $Q.\text{removeMin}()$ 
16:   if node  $u \in mst$  and node  $v \in mst$  then
17:     // TODO: What goes here?
18:   else
19:     // TODO: What about here?
20:   end if
21: end while
22: return  $mst$ 
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**Algorithm 2** Prim's algorithm with node priority queue

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1: Input: Graph  $G = (V, E)$ 
2: Output: Minimum spanning tree of  $G$ 
3:
4: // Initialization
5: Select an arbitrary vertex  $s \in V$ 
6: Construct an empty tree  $mst$ 
7: Construct an empty priority queue  $Q$  that will contain nodes ordered by their "distance" from
    $mst$  // If  $v \notin mst$ , then the distance of  $v$  is defined as the weight of the minimum cost edge
    $(u, v)$  such that  $u \in mst$ 
8: Insert  $s$  into  $Q$  with priority 0
9:
10: // Construct  $mst$ 
11: while there exists a vertex  $v$  s.t.  $v \in V$  and  $v \notin mst$  do
12:   Let  $v = Q.\text{findMin}()$ 
13:    $Q.\text{removeMin}()$ 
14:   for vertex  $u \in \text{neighbors}(v)$  do
15:     if  $v \notin mst$  then
16:       if  $\text{weight}(u, v) < Q.\text{getPriority}(u)$  then
17:         // TODO: What goes here?
18:       end if
19:     end if
20:   end for
21: end while
22: return  $mst$ 
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**Algorithm 3** Kruskal's algorithm: high level

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1: Input: Graph  $G = (V, E)$ 
2: Output: Minimum spanning forest of  $G$ 
3: Create a forest  $msf$ , initialized so that every vertex  $v \in V$  is a singleton tree in  $msf$ 
4: Create a set  $remainingedges$  containing all the edges in the graph
5: while  $remainingedges \neq \emptyset$  and  $msf$  is not spanning do
6:   Remove an edge  $(u, v)$  with minimum weight from  $remainingedges$ 
7:   if  $(u, v)$  connects two different trees in  $msf$  then
8:     Add  $(u, v)$  to  $msf$ , combining the two trees
9:   end if
10: end while
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**Algorithm 4** Kruskal's algorithm: with priority queue over edges

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```
1: Input: Graph  $G = (V, E)$ 
2: Output: Minimum spanning forest of  $G$ 
3:
4: // initialize forest
5: Create an empty map  $msf$  whose keys are nodes and values are trees
6: for vertex  $v \in V$  do
7:    $msf[v] \leftarrow$  tree with  $v$  as only node
8: end for
9:
10: // initialize priority queue
11: Create an empty priority queue  $Q$  whose elements are edges and priorities are weights
12: for edge  $(u, v) \in E$  do
13:   Insert  $(u, v)$  into  $Q$ 
14: end for
15:
16: // main loop
17: while  $|Q| > 0$  and  $msf$  is not spanning do
18:    $(u, v) \leftarrow Q.findMin()$ 
19:    $Q.removeMin()$ 
20:   if  $msf[u] \neq msf[v]$  then
21:     // TODO: What goes here?
22:   end if
23: end while
24: return  $msf$ 
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