1. (5.5) Consider two edge-weighted, connected graphs G and G'. The graphs have the same vertices and edges, but not necessarily the same edge weights.

   (a) Find a counterexample to the statement: G and G' must have the same minimum spanning tree.

   (b) Find a counterexample to the statement: G and G' cannot have the same minimum spanning tree.

   (c) Let $w_G(e)$ denote the weight of edge e in graph G and $w_{G'}(e)$ the weight in G'. The graphs obey the property that $w_G(e) = w_{G'}(e) + 1$.

   Find a counterexample to the statement: the shortest path between any two vertices s and t must be the same in both G and G'.

2. (5.9) Find counterexamples to the following statements concerning the undirected, connected graph $G = (V, E)$.

   (a) If G has more than $|V| - 1$ edges, and there is a unique heaviest edge, then this edge cannot be part of a minimum spanning tree.

   (b) The shortest path tree computed by Dijkstra's algorithm is necessarily a minimum spanning tree.

   (c) The shortest path between two nodes is necessarily part of some minimum spanning tree.

3. (5.20) A *perfect matching* is a set of edges that touches each node exactly once. Give a linear-time algorithm that takes as input a tree and determines whether it has a perfect matching.

4. (5.22/23) You are given a graph $G = (V, E)$ with positive edge weights, and a minimum spanning tree $T = (V, E')$ with respect to these weights; you may assume G and T are given as adjacency lists. Now suppose the weight of a particular edge $e \in E$ is modified from $w(e)$ to a new value $\hat{w}(e)$. You wish to quickly update the minimum spanning tree T to reflect this change, without recomputing the entire tree from scratch. There are four cases. In each case give a linear-time algorithm for updating the tree.

   (a) $e \in E'$ and $w(e) > \hat{w}(e)$.

   (b) $e \in E'$ and $w(e) < \hat{w}(e)$.

   (c) $e \notin E'$ and $w(e) < \hat{w}(e)$.

   (d) $e \notin E'$ and $w(e) > \hat{w}(e)$.