

## CSCI046 Homework 2: Runtime Analysis

**DUE: Thursday, 6 February beginning of class**

Name: \_\_\_\_\_

**Collaboration policy:** You are allowed to use any resources you would like to complete this assignment, and you are encouraged to work in teams. Remember, learning the material is your responsibility, so collaborate in a way that will help you learn.

**Grading note:** Each of these problems contains several subproblems, and there are typically more subproblems than the point value of the problem. You will lose 1 point for each incorrect subproblem. If this would result in a negative score, then you get zero for the problem.

**Problem 1.** (3 points) Simplify the following expressions:

1.  $O\left((n^2 + n \log n)(n^3 + \log n)\right)$

2.  $\Omega\left((3.45n + n)(\log n^2)\right)$

3.  $\Theta\left(n(1 + \log n) + n^{3.2} + \log 2^n\right)$

**Problem 2.** (3 points) Complete each equation below by adding the symbol  $O$  if  $f = O(g)$ ,  $\Omega$  if  $f = \Omega(g)$ , or  $\Theta$  if  $f = \Theta(g)$ . The first row is completed for you as an example.

$f(n)$		$g(n)$
1	=	$O(n)$
$3n \log n$	=	$n^2$
1	=	$1/n$
$\log_2 n$	=	$\log_3 n$
$n^{42}$	=	$42^n$
$5 \cdot 10^{30}$	=	$\log n$
$\log n$	=	$\log(n^2)$
$2^n$	=	$3^n$
$n!$	=	$n^2$
$\log n$	=	$(\log n)^2$



**Problem 4.** (2 points) Answer the questions below based on the following python code:

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```
1 for i in range(n**2):
2     print('a')
3     for j in range(n/2):
4         print('b')
5         for k in range(int(math.sqrt(n))):
6             print('c')
7             print('b')
8         print('b')
9 for i in range(n):
10    print('b')
11 for i in range(100):
12    print('d')
13    print('a')
```

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1. What is the asymptotic number of times that the letter **a** will be printed? (Use  $\Theta$  notation.)
2. What is the asymptotic number of times that the letter **b** will be printed? (Use  $\Theta$  notation.)
3. What is the asymptotic number of times that the letter **c** will be printed? (Use  $\Theta$  notation.)
4. What is the asymptotic number of times that the letter **d** will be printed? (Use  $\Theta$  notation.)

**Problem 5.** (0.5 points extra credit) Prove the following:  $\log(n!) = \theta(n \log n)$ ,