

CSCI046 Notes: Runtime Analysis

1 Counting

Our goal is to count how many times something will happen in our code. This is often used as a proxy for how long it takes a program to run.

Example 1.

```
1 print('x')
2 print('x')
3 print('x')
4
5 for i in range(10):
6     print('y')
7
8 for i in range(10,20):
9     print('z')
10    print('z')
11    print('z')
```

1. What is the exact number of times that the letter x will be printed?

2. What is the exact number of times that the letter y will be printed?

3. What is the exact number of times that the letter z will be printed?

Example 2. Answer the questions below based on the following python code:

```
1 for i in range(10):
2     print('x')
3 for i in range(20):
4     print('x')
5
6 for i in range(10):
7     for j in range(20):
8         for k in range(30):
9             print('y')
10
11 print('z')
12 for i in range(10):
13     print('z')
14     for j in range(10):
15         print('z')
16         print('z')
17     for j in range(10):
18         print('z')
19 for i in range(10):
20     print('z')
```

1. What is the exact number of times that the letter x will be printed?

2. What is the exact number of times that the letter y will be printed?

3. What is the exact number of times that the letter z will be printed?

Example 3. Answer the questions below based on the following python code:

```
1 for i in range(n):
2     print('x')
3 for i in range(n*2):
4     print('x')
5
6 for i in range(n):
7     for j in range(n*2):
8         for k in range(n*3):
9             print('y')
10
11 print('z')
12 for i in range(n):
13     print('z')
14     for j in range(n):
15         print('z')
16         print('z')
17     for j in range(n):
18         print('z')
19 for i in range(n):
20     print('z')
```

1. What is the exact number of times that the letter x will be printed?
2. What is the exact number of times that the letter y will be printed?
3. What is the exact number of times that the letter z will be printed?

Example 4. Answer the questions below based on the following python code:

```
1 for i in range(n):
2     for j in range(0,i):
3         for k in range(0,j):
4             print('x')
5
6 for i in range(n):
7     for j in range(i,n):
8         for k in range(j,n):
9             print('y')
10
11 for i in range(n):
12     for j in range(i,n):
13         for k in range(i,j):
14             print('z')
```

1. What is the exact number of times that the letter x will be printed?
2. What is the exact number of times that the letter y will be printed?
3. What is the exact number of times that the letter z will be printed?

2 Math: Big-O/ Θ / Ω Notation

Key Ideas:



Definition 1. Let f, g be functions from $\mathbb{R}^+ \rightarrow \mathbb{R}^+$. Then,

1. If $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} < \infty$, then we say $f = O(g)$.
2. If $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} > 0$, then we say $f = \Omega(g)$.
3. We say that $f = \Theta(g)$ if both $f = O(g)$ and $f = \Omega(g)$.

Intuitively, you should think of O as \leq , Ω as \geq , and Θ as $=$.

Example 5.

1. $f(x) = x$
 $g(x) = x^2$

2. $f(x) = x^2$
 $g(x) = x$

3. $f(x) = x^2 + 2x + 5$
 $g(x) = x$

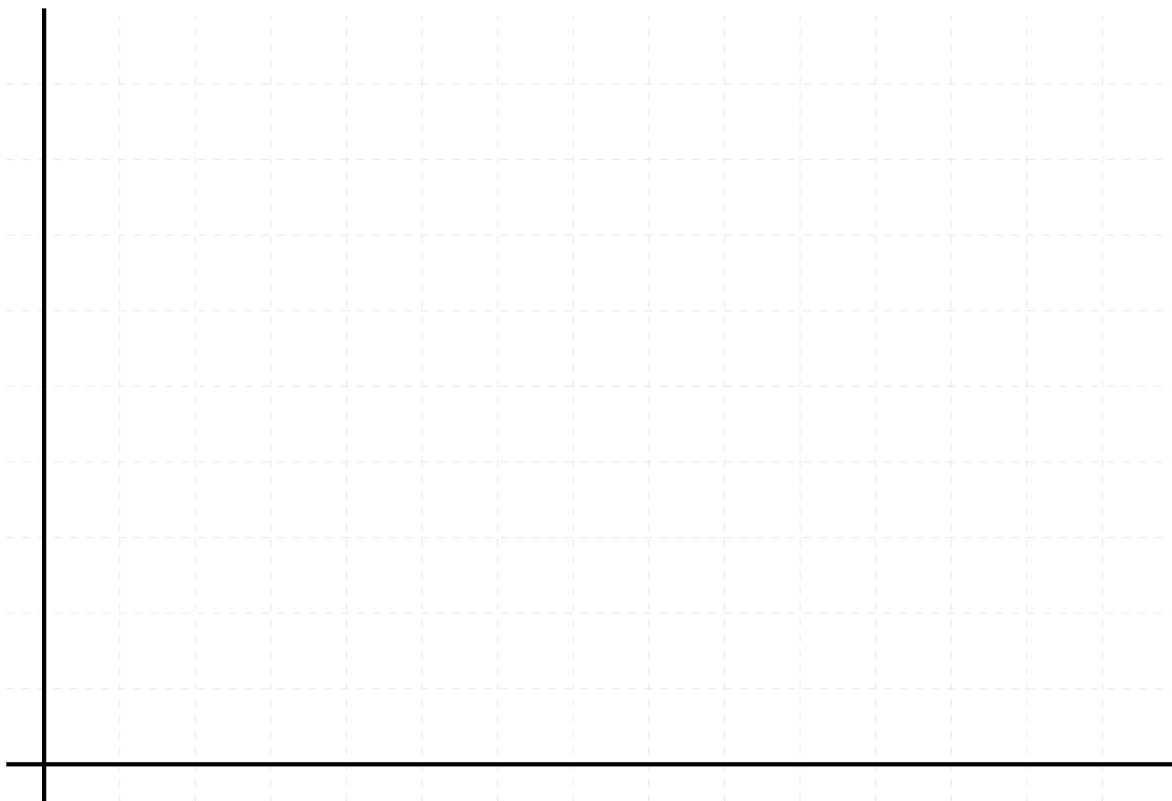
4. $f(x) = x^2 + 2x + 5$
 $g(x) = x^2$

5. $f(x) = x^2 + 2x + 5$
 $g(x) = x^3$

Example 6. What happens when we double the size of the inputs?

Example 7. You should memorize the relationship between the following functions:

1 $\log n$ n $n \log n$ n^2 n^3 2^n



Example 8. Complete each equation below by adding the symbol O if $f = O(g)$, Ω if $f = \Omega(g)$, or Θ if $f = \Theta(g)$. The first row is completed for you as an example.

$f(n)$		$g(n)$
1	=	$O(n)$
$3n \log n$	=	n^2
1	=	$1/n$
$\log_2 n$	=	$\log_3 n$
$\log n$	=	$\frac{1}{\log n}$
$5 \cdot 10^{30}$	=	$\log n$
$\log n$	=	$\log(n^2)$
2^n	=	3^n
$\frac{1}{n}$	=	$\sqrt{\frac{1}{n}}$
$\log n$	=	$(\log n)^2$

Example 9. Simplify the following expressions:

1. $O\left(n^3 + n^2\right)$

2. $O\left(n^3 + 5n^2 \log n + \log n\right)$

3. $O\left(100000000000\right)$

4. $O\left(\log n + 100000000000\right)$

5. $O\left(\frac{1}{n} + \frac{1}{n^2}\right)$